Probability of moraine survival in a succession of glacial advances

Anthony B. Gibbons
U.S. Geological Survey, P.O. Box 25046, Federal Center, Denver, Colorado 80225
Joe. D. Megeath
Department of Computer and Management Science, Metropolitan State College, Denver, Colorado 80204
Kenneth L. Pierce
U.S. Geological Survey, P.O. Box 25046, Federal Center, Denver, Colorado 80225

ABSTRACT

Emplacement of glacial moraines normally results in obliteration of any older moraines deposited by less extensive glacial advances, a process we call "obliterative overlap." A probability analysis of the likely impact of obliterative overlap on the completeness of the glacial record assumes that moraines were deposited at various distances from their glacial source areas randomly over time. Assuming randomness and obliterative overlap, after 10 glacial episodes, the most likely number of surviving moraines is only three. The record of the Pleistocene is in agreement with the probability analysis: the 10 glaciations during the past 0.9 m.y. inferred from the deep-sea record resulted in moraine sequences in which only two or three different-aged moraine belts can generally be distinguished.

INTRODUCTION

The record of glaciation as defined by moraines is a selective one because glacial advances generally obliterate moraines deposited by less extensive previous glacial advances. We use the term "obliterative overlap" to define this process of obliteration of a moraine when overlapped by a later, more extensive glacial advance. The result of this process is a succession of moraines that are progressively younger in the direction of the source area (see Fig. 1) and give little clue as to whether they actually include all the moraines that were deposited. We here develop a probability theory that can be applied to this kind of sequence, assuming that the ranking with respect to distance of the glacial advances was random over time.

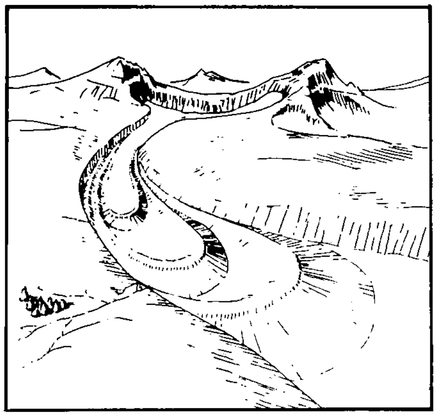

Figure 1. Diagram showing typical sequence of moraines preserved in a mountain valley, from oldest and farthest advanced to youngest and least advanced. Moraines surviving in surface succession were deposited by glaciations more extensive than all subsequent glaciations.

Figure 2 defines the relation between glacial events and deposits as used in this paper.

PROBABILITY ANALYSIS

The following probability model is an analysis of all possible outcomes under the following assumptions: (1) The ranks attained by moraines as a result of the relative distances of glacial advances are random over time. (2) Moraines of a succession are restricted to the same glacial pathway. (3) The only moraines to survive obliterative overlap are those deposited at distances greater than any subsequent moraine.

The assumption of random positional rank was chosen because of its simplicity and its applicability to periods in which no overall trend in extent of glaciations can be inferred. Such periods include intervals of time long enough to span several glacial-interglacial advances and perhaps also the culmination period of a major glacial advance.

The analysis of concern here is the probability of n surviving moraines under the condition that N glacial episodes took place. The probability notation $P(n|N)$, where n = number of surviving moraines, N = number of glacial episodes, and $P(n|N)$ is the probability of n surviving moraines under the condition that N glacial episodes took place will be used for the remainder of this analysis. Given the three assumptions listed above, n must be less than or equal to N. It is also apparent that $P(1|1) = 1.000$. Similarly, if the last (youngest) glacier ranks the longest, there will be only one surviving moraine. Therefore, from the three assumptions made for the purposes of this analysis, $P(1|N) = 1/N$. For example, if there were four glacial episodes and the moraines produced during the four episodes were random in rank, then the highest ranking of the four has a 1/4 chance of being the latest. Hence, in this case $P(n|N) = P(1|4) = 1/4$.

A significant special case of this analysis is the probability that the number of surviving moraines is the same as the number of glacial episodes. Again, four glacial episodes will be used as an example. In the case $N = 4$, all four moraines will survive only if the oldest moraine is at the greatest distance, the second oldest is at the second greatest, and so on. The probability that four glacial episodes would leave four surviving moraines is $P(4|4) = 1/4 \times 1/3 \times 1/2 \times 1 = 1/24 = 0.042$.

In general, the special case of the number of moraines surviving from different numbers of glacial episodes being equal to the number of glacial episodes is given by $P(n|N) = 1/N!$, where $n = N$.

Continuing with the example of four glacial episodes ($N = 4$), it is first recognized that $4! = 24$ different orderings of moraine distances could occur. The notation for listing these orders will be (1) the order in time listed from left to right, (2) the order in distance from source indicated by rank, where 1 is the most

EVENT		DEPOSIT
Glacial episode	Advance	Moraine*
	Glaciation	Moraine belt*

* Here includes all till at the surface whether or not it has morainal form.

Figure 2. Nomenclature used in this paper, showing relation between glacial events and glacial deposits.

distant, and (3) indication of survivors by underscoring. For example, the listing 4 3 2 $\underline{1}$ indicates that the lowest ranking (least distant) moraine (4) occurred first, the next lowest (3) occurred second, the next above that in rank (2) occurred third, and the highest ranking (1), the most distant, occurred last. This would result in one surviving moraine, because the most distant or advanced (1) would erase the previous three.

Out of the exhaustive list of 24 possible combinations, the following 11 combinations are those that result in exactly two surviving moraines.

$4 \quad 3 \quad \underline{1} \quad \underline{2}$
$3 \quad 4 \quad \underline{1} \quad \underline{2}$
$4 \quad \underline{1} \quad 3 \quad \underline{2}$
$3 \quad \underline{1} \quad 4 \quad \underline{2}$
$\underline{1} \quad 4 \quad 3 \quad \underline{2}$
$\underline{1} \quad 3 \quad 4 \quad \underline{2}$
$2 \quad \underline{1} \quad 4 \quad \underline{3}$
$2 \quad 4 \quad \underline{1} \quad \underline{3}$
$4 \quad 2 \quad \underline{1} \quad \underline{3}$
$2 \quad 3 \quad \underline{1} \quad \underline{4}$
$3 \quad 2 \quad \underline{1} \quad \underline{4}$

Consequently, P(2|4) = 11/24 = 0.4583.

The probability P(2|4) is an instance of the general case, as distinct from the special cases of one surviving moraine and all moraines surviving that were examined above. In the general case, the probability of *n* surviving moraines from *N* glacial episodes is given by

$$P(n|N) = 1/N \sum_{N=N-1}^{N-1} P(n-1|N),$$

for $N \geqslant n \geqslant 2$.

As an example of a simple computation using the general formula, we again evaluate P(2|4):

$$P(2|4) = 1/4 \sum_{N=1}^{3} P(1/1/N) =$$

$$1/4[P(1|1) + P(1|2) + P(1|3)].$$

Evaluating, P(1|1) = 1, P(1|2) = 1/2, and P(1|3) = 1/3 [all from P(1|*N*) = 1/*N*], and P(1|3) = 1/3

$$P(2|4) = 1/4 = 1/4(1 + 1/2 + 1/3) = 1/4(6/6 + 3/6 + 2/6) = 1/4(11/6) = 11/24 = 0.4583.$$

TABLE 1. PROBABILITIES OF *n* MORAINES GIVEN *N* GLACIAL EPISODES

N	*n*=1	2	3	4	5	6	7	8	9	10
1	1.0000	----	----	----	----	----	----	----	----	----
2	0.5000	0.5000	----	----	----	----	----	----	----	----
3	0.3333	0.5000	0.1667	----	----	----	----	----	----	----
4	0.2500	0.4583	0.2500	0.0417	----	----	----	----	----	----
5	0.2000	0.4167	0.2917	0.0833	0.0083	----	----	----	----	----
6	0.1667	0.3806	0.3125	0.1181	0.0208	0.0014	----	----	----	----
7	0.1429	0.3500	0.3222	0.1458	0.0347	0.0042	0.0002	----	----	----
8	0.1250	0.3241	0.3257	0.1679	0.0486	0.0080	0.0007	0.0000	----	----
9	0.1111	0.3020	0.3255	0.1854	0.0619	0.0125	0.0015	0.0001	0.0000	----
10	0.1000	0.2829	0.3232	0.1994	0.0742	0.0174	0.0026	0.0002	0.0000	0.0000
15	0.0667	0.2168	0.2999	0.2378	0.1221	0.0433	0.0110	0.0021	0.0003.	0.0000
20	0.0500	0.1774	0.2748	0.2508	0.1527	0.0664	0.0215	0.0053	0.0010	0.0002

Note: Probabilities of survival of moraines in a sequence assuming ranking of moraine distance from source area is random through time. *N* = 10 is the number of glacial-interglacial oscillations in the last 0.9 m.y.

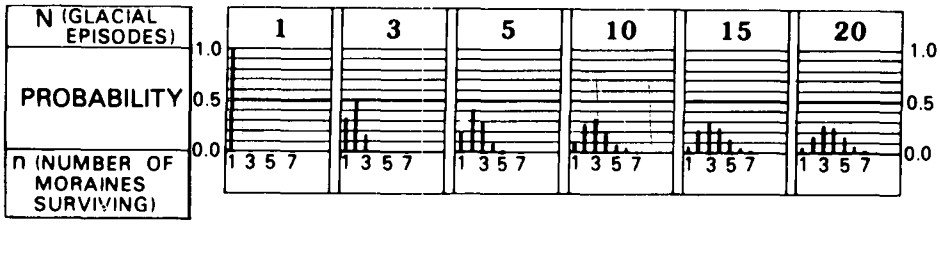

Figure 3. Graph indicating probability of survival of moraines (*n*) after 1, 3, 5, 10, 15, and 20 glacial episodes (*N*). Probabilities of less than about 0.01 are not discernible on plot, but are listed in Table 1.

Because the probability structure is built up in a stepwise manner, calculations at higher levels of *n* and *N* become laborious. A computer program was used to generate Table 1, which lists the probability of *n* surviving moraines in the range 1–20 glacial episodes.

After the first few glacial episodes, the expectation of larger numbers of surviving moraines increases much more slowly than the number of glacial episodes (Table 1; Fig. 3). Figure 4 clearly illustrates this nonlinearity between *N* and *n* and shows that a record consisting of three or fewer moraines is a strong probability whether or not the number of glacial episodes was comparably small.

PLEISTOCENE GLACIAL RECORDS

Because our probability model applies only to cases where distance rank of moraines may be assumed to be random through time, this analysis is limited to situations where no dominant trend can be inferred. Part A of Figure 5 diagrammatically shows a single glaciation beginning with an advancing trend (A1) in which all moraines would be systematically obliterated by later and farther ice advances. The retreat trend (A3) imparts a nonrandom bias toward survival of moraines. However, if ranking of glacial advances during a glacial culmination period (A2) can be assumed to be random, probability analysis is applicable to moraine survival. Where only *n* is known, the analysis functions as a *guide* to an unknown—the actual number of ice advances (*N*) during the culmination period of a glaciation.

Parts B1 and B2 of Figure 5 show two periods of random variation about a mean approximated during successive culminations, but the largest culminations of B1 were exceeded by later culminations, and no moraines of B1 will survive. Part B2 corresponds to glacial-interglacial cycles of the last half of the Pleistocene. For this period, we think we have a reasonable basis for estimating both *N* (gla-

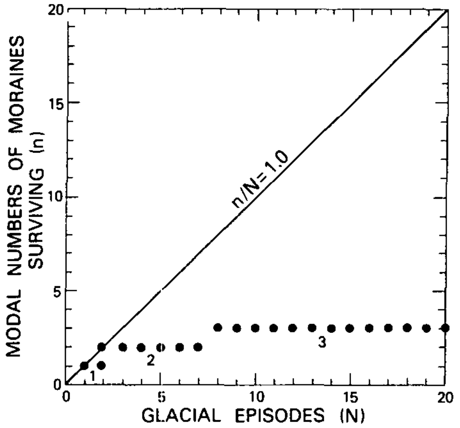

Figure 4. Plot of glacial episodes (*N*) vs. most likely number of surviving moraines (*n*).

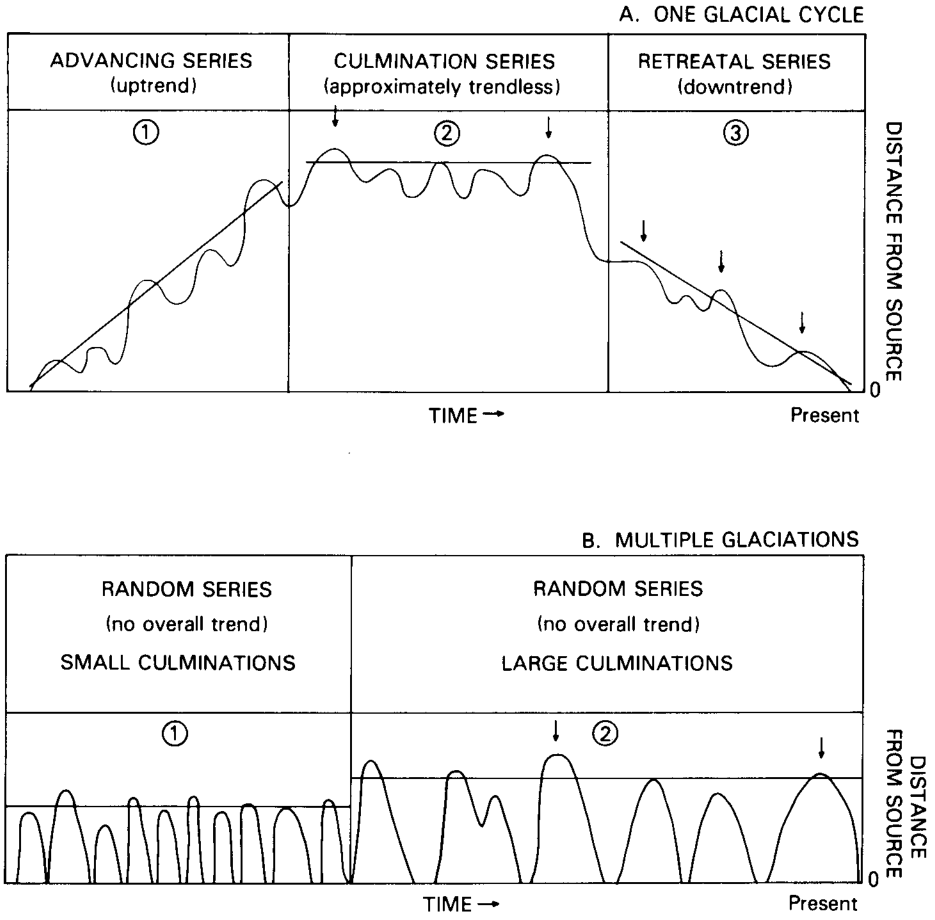

Figure 5. Idealized plots of distance of ice front from its source over time. Horizontal lines approximate overall magnitude of culminations. Small arrows indicate culminations that would leave surviving moraines. A: Phases of one glacial cycle. B: Sequence of glaciations (only first-order oscillations shown).

ciations) and *n* (moraine belts). The probability analysis functions as an explanation of the relation between *n* and *N*. Its explanatory power in the following *test* reflects directly on its credibility as a guide to *N* (number of glacial episodes) in those cases where there is no independent estimate of *N*.

The best available indicator of the sequence of global glacial-interglacial oscillations during the Pleistocene comes from analysis of ^{18}O and other constituents of deep-sea cores (Shackleton and Opdyke, 1973, 1976; van Donk, 1976), although deep-sea cores actually provide only a proxy record of the actual extent of glacial ice. Intervals of relatively high ^{18}O result from the combined effects of buildup of ^{18}O-poor glaciers on land and cooler water temperatures, although a precise physical explanation of the ^{18}O record has yet to be established. The ^{18}O record from many cores shows similar pattern and chronology and parallels other glacial-interglacial proxy records, such as those of foraminiferal assemblages, CaCO$_3$ content, and ice-rafted clasts (e.g., Kukla, 1977; Cline and Hays, 1976; Andrews, 1982, for discussion of problems with ^{18}O record).

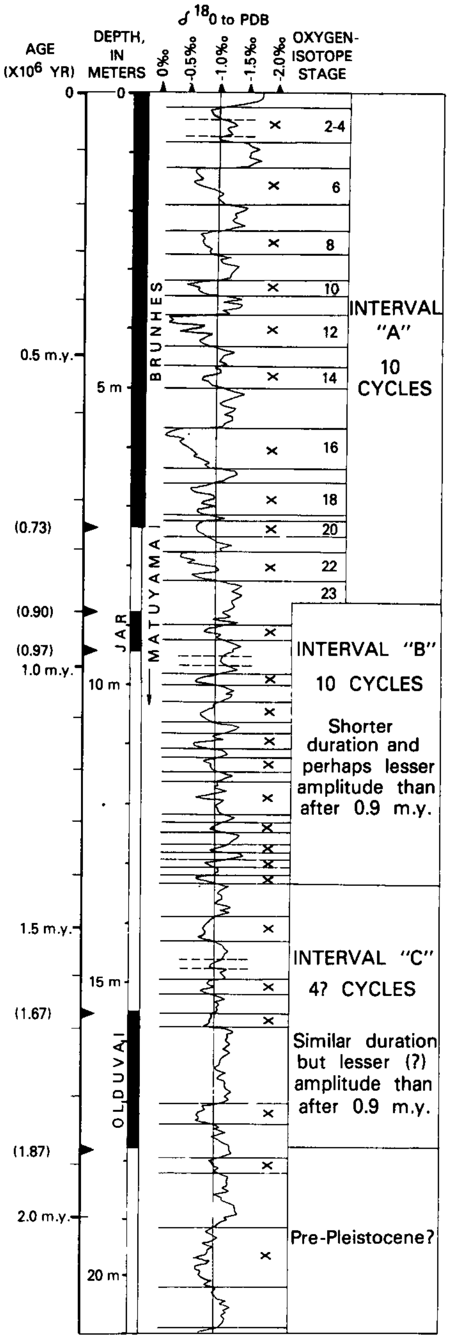

Figure 6. Oxygen-isotope record for entire Pleistocene from core V28-239 from western equatorial Pacific Ocean (Shackleton and Opdyke, 1976). Number of ^{18}O cycles given for Pleistocene intervals "A," "B," and "C." Determination of *N* is provisionally limited to 10 cycles during past 0.9 m.y. because older glacial-interglacial cycles may be of systematically lesser amplitude. Ages for paleomagnetic boundaries from Mankinen and Dalrymple (1979). Time scale for Brunhes from Shackleton and Opdyke (1976), adjusted so base of Brunhes is 0.73 Ma. Base of Pleistocene assumed to be in Olduvai paleomagnetic event (see Obradovich et al., 1982).

As shown in Figure 6, interval "A" from 0 to 0.9 m.y. has a trendless-appearing set of 10 major glacial-interglacial oscillations. Kominz and Pisias (1979) concluded that climatic fluctuation over the past 730,000 yr (Brunhes) was mainly stochastic. Available evidence suggests that glacial culminations may have been systematically less in intervals "B" and "C" (Fig. 6), and therefore *N* is here provisionally limited to 10, the number of glacial-interglacial oscillations in the past 0.9 m.y. (van Donk, 1976; Ronai, 1969, as dated by Cooke et al., 1979).

The time span of *N* cannot be greater than the time required for the removal of all deposits of a glaciation by normal processes of erosion. Glacial deposits older than the 0.7-m.y.-old Bishop Tuff are extensively preserved at the eastern foot of the Sierra Nevada, and glacial deposits older than the 0.6-m.y.-old Lava Creek ash are common in the western mid-continental United States. Thus, total removal by normal erosion is not expectable for deposits less than 0.9 m.y. old, and *N* = 10 appears realistic.

In applying the analysis at the level of glaciations, *n* refers to the number of moraine belts that have survived by virtue of not having been entirely overlapped during a later glaciation. The term "moraine belt" designates all surviving moraines of a given glaciation. In the assessment of *n*, three qualifications apply: (1) Moraines in areas with major Quaternary changes in altitude due to volcanism or large imbalance between uplift and erosion are excluded because such changes would introduce an overall trend toward increasing or decreasing extent of glaciation. (2) Moraines such as the type Mono Basin moraines of the eastern Sierra Nevada that survive solely because glacial flow in subsequent glaciations was not in the same direction violate the second basic assumption and are excluded. (3) All deposits of the *last* glaciation count as 1 in *n*, because they are elements of the same moraine belt deposited during a 1 glaciation as represented in the count of *N* (Fig. 2).

Almost all values of *n* lie between 1 and 4. Locally, $n = 1$, as the Okanagan lobe of the Cordilleran ice sheet (Richmond et al., 1965) and the Cataract Creek and North Willow Creek valleys of the Tobacco Root Mountains, Montana (Hall and Heiny, 1983). For the Kittitas valley, Washington, *n* may equal 4 or perhaps 5 (Porter, 1976).

In most cases, $n = 2$ or 3 moraine belts. In the Rocky Mountains in addition to deposits of the last glaciation (Pinedale), moraines from the preceding glaciation (Bull Lake) are common, and deposits of still older glaciations (pre–Bull Lake) are preserved in probably a minority of cases. Here, $n = 2$ or 3, depending on the presence or absence of pre–Bull Lake moraines. [See Richmond (1965) for listing of moraines and Pierce (1979) and Colman and Pierce (1981) for evidence that some of the deposits assigned to the Bull Lake glaciation are separated by a full interglacial from the Pinedale and thus represent a separate moraine belt at the "glaciation" level.] For most valleys in the Alps, $n = 2$ or 3, assuming that the Riss glaciation predates the last glaciation (see reports in Richmond, 1968). For most lobes along the southern margin of the Laurentide and Scandinavian ice sheets, maps of different-aged glacial deposits show that *n* is commonly 2 or 3 moraine belts for the mid-continental United States and 2 to 4 for northern Europe and England (e.g., Flint, 1971, p. 545, 594; Goldthwait et al., 1965; Lineback, 1979; Kukla, 1977, Fig. 19).

On the basis of the probability analysis, if $N = 10$, the most likely value of *n* is 3 (32%), followed by $n = 2$ (28%), $n = 4$ (20%), and $n = 1$ (10%), with a 90% chance that $n = 1$ to 4. These predicted probabilities are quite close to the range of 1 to 4 and the most commonly observed values of 2 to 3 moraine belts represented in the deposits of a given glaciated valley or ice-sheet lobe.

CONCLUSIONS

The characteristic aspect of younger sourceward or retreat of moraine sequences need not reflect anything more than the way such series accumulate. The number of moraine belts noted in various areas of the world does not constrain an inference as to the total number of Pleistocene glaciations, other than that there have been at least as many glaciations as moraine belts. The presence of, say, three moraines representing deposits of a given glaciation in one area does not necessarily mean that the same glacial advances are represented in a three-moraine sequence elsewhere. If glacial advances are of similar magnitude (Fig. 5, part A), small climatic or topographic differences between areas may cause changes in rank and consequent survival patterns of moraines. This could occur without changing the *number* of moraines that survive. This same general reasoning also applies at the level of moraine belts and glaciations (see Fig. 5, part B). Moraines that are unusual in that they do not correlate with "standard" sequences are of particular value because they fill gaps in the Pleistocene chronology. The numerical mismatch between the typical 2–3 moraine belts of the global Pleistocene and the number of major Pleistocene glacial-interglacial oscillations—probably 10, but perhaps as many as 24—matches the expected outcome of the process of obliterative overlap, functioning in a random manner over time.

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